On a q-deformation of the supersymmetric Witten model

1993 J. Phys. A: Math. Gen. 26 7219

(http://iopscience.iop.org/0305-4470/26/23/055)

J. Phys. A: Math. Gen. 26 (1993) 7219–7226. Printed in the UK

On a q-deformation of the supersymmetric Witten model

N Debergh†

Theoretical and Mathematical Physics, Institute of Physics, B.5, University of Liège, B-4000-LIEGE 1, Belgium

Received 7 June 1993, in final form 23 August 1993

Abstract. An introduction to a q-superspace formulation is performed, leading to deformed superfields. The corresponding free and interacting Lagrangians (Hamiltonians) are constructed in the classical as well as quantum contexts. In particular, a q-deformation of the supersymmetric Witten Hamiltonian is derived.

1. Introduction

Supersymmetric quantum mechanical systems are characterized by the existence of N supercharges, each of them being the square root of the corresponding Hamiltonian. They have been intensively studied in the recent literature ([1] and references therein), specific attention having been paid to the case $N=2$. In particular, their relationship with supersymmetric field theory [2] has been exploited by constructing supersymmetric Lagrangians through the superfield method. Indeed, it is well known that, from real superfields [3] and by making use of the superspace formulation [4], one can consider a supersymmetric Lagrangian leading to the $N=2$-supersymmetric Witten model [5], i.e. to the usual supersymmetric quantum mechanics.

Moreover, for the past few years, another generalization of quantum mechanics has been proposed. This extension, based on quasi-triangular Hopf algebras [6] is referred to as the so-called quantum deformation and, together with its generalization to field theory, has found many applications in physics [7] (conformal field theories, integrable systems, etc). In this spirit, a deformation of Grassmann variables has been recently pointed out [8], when the deformation parameter q has been restricted to the roots of unity. We plan here to exploit and adapt this proposal in order to combine these two above-mentioned extensions of quantum mechanics and construct a deformation of the $N=2$-supersymmetric Witten Hamiltonian.

The contents of this paper are then arranged as follows. In section 2, we give an outlook of a deformed superspace formulation and construct the corresponding generalized superfields. We then propose a free deformed supersymmetric Lagrangian, in the classical as well as quantum contexts (sections 3.1 and 3.2, respectively). Section 4 is devoted to the extension of this Lagrangian to the interacting case. We conclude in section 5 with some comments. Our conventions consist in putting $\hbar=1$ and $m=1$.

† Chercheur IISN, Brussels.

0305-4470/93/237219+08$07.50

2. A q-superspace formulation

As already mentioned by Filippov *et al* [8], Grassmann variables can be extended to the q-context by imposing

$$\theta_j^k = 0 \quad \text{if } q^k = 1 \qquad k > 2 \tag{2.1}$$

and

$$\theta_i\theta_j = q\theta_j\theta_i \qquad i<j \qquad i,j = 1,2,\ldots,N \tag{2.2}$$

where k and N are positive integers. Let us restrict ourselves to the values $N=2$ and $k=3$ and let us choose the corresponding Grassmann variables to be real ones, this last option being compatible with (2.2). Let us also introduce real derivatives with respect to these Grassmann quantities. They satisfy

$$\partial_{\theta_j}^3 = 0 \qquad \partial_{\theta_i}\partial_{\theta_j} = q\partial_{\theta_j}\partial_{\theta_i} \qquad i<j \tag{2.3}$$

and

$$\partial_{\theta_i}\theta_j = q\theta_j\partial_{\theta_i} + \mathrm{i}q^{1/2}\delta_{ij} \tag{2.4}$$

in agreement with the reality conditions. We then propose the following q-deformed covariant derivatives

$$D_j = \partial_{\theta_j} + \mathrm{i}\theta_j^2\partial_t \qquad j = 1,2 \tag{2.5}$$

such that

$$D_j^3 = \mathrm{i}\partial_t \tag{2.6}$$

with

$$\partial_{\theta_j}\theta_i^2 = q^2\theta_i^2\partial_{\theta_j} - \mathrm{i}q^{-1/2}\theta_i\delta_{ij}. \tag{2.7}$$

Notice that the derivatives (2.5), although differing from previously defined ones ([8, 9]), lead to the same particular consequence, i.e. a dimension $L^{1/3}$ associated with the deformed Grassmann quantities θ_j.

We can also define some new specific q-deformed superfields such as

$$\begin{aligned}\Phi = x(t) + q^{-1/2}\theta_1\psi_1(t) + q^{-1/2}\theta_2\psi_2(t) + q^{-1/2}\theta_1^2A_1(t) + q^{-1/2}\theta_2^2A_2(t)\\ + \theta_1\theta_2B(t) + q\theta_1^2\theta_2\chi_1(t) + q^2\theta_2^2\theta_1\chi_2(t) + \theta_1^2\theta_2^2F(t).\end{aligned} \tag{2.8}$$

It is easy to convince ourselves that such a particular field is a real one through the conventions (no summation on repeated indices)

$$\theta_j\psi_j = q\psi_j\theta_j \qquad \theta_2\psi_1 = q\psi_1\theta_2 \qquad \psi_2\theta_1 = q\theta_1\psi_2 \tag{2.9a}$$

$$A_j\theta_j = q\theta_jA_j \qquad A_1\theta_2 = q\theta_2A_1 \qquad \theta_1A_2 = qA_2\theta_1 \tag{2.9b}$$

$$B\theta_1 = \theta_1B \qquad B\theta_2 = q\theta_2B \tag{2.9c}$$

$$\chi_j\theta_1 = \theta_1\chi_j \qquad \chi_1\theta_2 = q\theta_2\chi_1 \qquad \chi_2\theta_2 = q\theta_2\chi_2 \tag{2.9d}$$

$$F\theta_1 = \theta_1F \qquad qF\theta_2 = \theta_2F \tag{2.9e}$$

while the spatial variable $x(t)$ commutes with the q-Grassmann quantity θ_1 and is such that $x\theta_2 = q\theta_2x$. The nine real fields can be classified into three sets: the 0-sector (x, χ_j), the 1-sector (B, A_j) and the 2-sector (F, ψ_j), leading to a Z_3-graduation [10].

3. A free q-deformed supersymmetric Lagrangian and the corresponding Hamiltonian

Let us now propose specific free q-deformed supersymmetric Lagrangians in the classical as well as quantum contexts (sections 3.1 and 3.2, respectively).

3.1. The classical case

Let us define the free q-supersymmetric Lagrangian by analogy with the undeformed case [3] (corresponding to $q=1$) through

$$\mathscr{L}_0=\mathscr{C}(q)(\bar{D}\Phi)(D\Phi) \tag{3.1}$$

where

$$D=(D_1-\mathrm{i}D_2)/\sqrt{2} \qquad \bar{D}=(D_1+\mathrm{i}D_2)/\sqrt{2}. \tag{3.2}$$

The function $\mathscr{C}(q)$ is present in this context in order to ensure the self-adjointness of the forthcoming Lagrangian. A key point is also to notice that the only significant terms are those multiplied by $\theta_1^2\theta_2^2$, a direct extension of the supersymmetric case [3] which points out the terms in $\theta_1\theta_2$, as can be verified through integration on Grassmann variables [4] or via the so-called projection technique [11]. An integration with respect to q-Grassmann quantities having not been clearly defined, we choose here to adapt the projection technique by noticing that

$$D^2\bar{D}^2\theta_1^2\theta_2^2R(t)|_{\theta_1=\theta_2=0}=-\tfrac{1}{4}qR(t) \tag{3.3}$$

whatever the field $R(t)$ is.

Thus, by defining

$$C(q)=-\tfrac{1}{4}q\mathscr{C}(q) \tag{3.4}$$

and by using the relations (2.1)–(2.4), (2.7) and (2.9), we are led to the following free Lagrangian

$$\begin{aligned} L_0=D^2\bar{D}^2\mathscr{L}_0|_{\theta_1=\theta_2=0}&=L_0(\dot{x},\psi_j,\dot{\psi}_j,A_j,\dot{A}_j,B,\chi_j,F)\\ &=\tfrac{1}{2}C(q)[-2q^{-1/2}\dot{x}\chi_1-2q^{-1/2}\dot{x}\chi_2-q\chi_1^2-q\chi_2^2+\mathrm{i}q^{1/2}\psi_1\dot{A}_1\\ &\quad-\mathrm{i}q^{3/2}\dot{A}_1\psi_1+\mathrm{i}q^{-1/2}\dot{A}_2\psi_2-\mathrm{i}q^{-1/2}\psi_2\dot{A}_2-\mathrm{i}q^{-1/2}A_1\dot{\psi}_1\\ &\quad+\mathrm{i}q^{-1/2}\dot{\psi}_1A_1-\mathrm{i}q^{1/2}\dot{\psi}_2A_2+\mathrm{i}q^{3/2}A_2\dot{\psi}_2]. \end{aligned} \tag{3.5}$$

We have imposed that

$$\psi_1A_2=-qA_2\psi_1 \qquad \psi_2A_1=-A_1\psi_2 \tag{3.6a}$$

$$B\psi_2=-\psi_2B \qquad B\psi_1=-q\psi_1B \tag{3.6b}$$

$$A_1F=-qFA_1 \qquad FA_2=-qA_2F. \tag{3.}$$

One can make sure that the algebra (3.6) (together with the following (4.5)) is ass ative by noticing that, for example, we have

$$(\psi_1\psi_2)B=\psi_1(\psi_2B)=qB\psi_2\psi_1.$$

It is also easy through (3.6) to convince ourselves that L_0 will be self-adjoint iff

$$C(q)=\alpha q^2+\mathrm{i}\beta(1-q) \tag{3.7}$$

α and β being real numbers.

A simple look at the Lagrangian (3.5) shows that there are four *auxiliary* fields (B, F, χ_j). Two of them simply vanish

$$B=F=0 \tag{3.8}$$

while the other two, by satisfying the Euler–Lagrange equations $\partial L_0/\partial\chi_j=0$, lead to

$$\chi_1=-q^{3/2}\dot{x} \tag{3.9a}$$

$$\chi_2=-q^{3/2}\dot{x}. \tag{3.9b}$$

Through these two results (3.9) and in accordance with some previously studied variations [10], we can thus propose the q-deformed free Hamiltonian of the form

$$\begin{aligned} H_0&=\dot{x}\frac{\partial L_0}{\partial\dot{x}}+\dot{\psi}_1\frac{\partial L_0}{\partial\dot{\psi}_1}+\dot{\psi}_2\frac{\partial L_0}{\partial\dot{\psi}_2}+\dot{A}_1\frac{\partial L_0}{\partial\dot{A}_1}+\dot{A}_2\frac{\partial L_0}{\partial\dot{A}_2}-L_0\\ &=\tfrac{1}{2}C(q)(2q)\dot{x}^2. \end{aligned} \tag{3.10}$$

With the particular choice (see (3.7))

$$\alpha=\tfrac{1}{2} \qquad \beta=0 \tag{3.11}$$

this operator (3.10) reduces to

$$H_0=\tfrac{1}{2}\dot{x}^2 \tag{3.12}$$

as expected.

3.2. *The quantum case*

With the specific context $q^3=1$, it is straightforward to see that the deformed bracket $[3]=0$ if, for example, we adopt the Macfarlane definition [12]

$$[n]=\frac{q^n-q^{-n}}{q-q^{-1}}. \tag{3.13}$$

This implies that the corresponding Fock space is generated by only three basis states $(|n\rangle, n=0, 1, 2)$ due to the nilpotency of the q-bosonic creation and annihilation operators [10]. The latter are characterized by

$$a|n\rangle=\sqrt{[n]}|n-1\rangle \qquad a^\dagger|n\rangle=[n+1]^{1/2}|n+1\rangle. \tag{3.14}$$

Moreover, it is well known [13] that, in this case, i.e. when q is root of unity, both operators x and p can be simultaneous self-adjoint ones. This possibility is expressed through

$$px-qxp=-\mathrm{i}\sqrt{q}. \tag{3.15}$$

One can thus identify, as usual, p with the following linear combination

$$p = \mathrm{i}\sqrt{\frac{\omega}{2}}\,(a^\dagger - a)$$

and obtain

$$p^3|n\rangle = 0. \tag{3.16}$$

Notice that, in accordance with (3.15) and $x^\dagger = x$, we obtain here

$$x = \frac{1}{1+q}\sqrt{\frac{2q}{\omega}}\,(q^N a + a^\dagger q^{-N})$$

where $N|n\rangle = n|n\rangle$.

The result (3.16) combined with the Heisenberg scheme gives rise to

$$\dot{x} = \mathrm{i}[H_0, x] = \frac{2q^{3/2}}{1+q}\,p + 2\mathrm{i}(q^2-1)xp^2 \tag{3.17}$$

and to the corresponding Hamiltonian (see (3.12))

$$H_0 = \frac{2q}{(1+q)^2}\,p^2. \tag{3.18}$$

We recover in these two operators previously published results from the Aref'eva–Volovich paper [13] in connection with the description of a quantum-free particle on the quantum line. Moreover, both operators (3.17) and (3.18) coincides with well known results when $q \to 1$ and, for the two other roots of unity, lead to

$$\dot{x}_{\mp} = (\mp 1 + \mathrm{i}\sqrt{3})(p - \sqrt{3}xp^2) \tag{3.19}$$

and

$$H_0 = 2p^2. \tag{3.20}$$

4. An interacting q-deformed supersymmetric Lagrangian and the corresponding Hamiltonian

Let us first concentrate once again on the classical context. The supersymmetric Witten model is based on the adding of an interacting term $W(\Phi)$ to the kinetic part, $W(x)$ (see (4.2)) referring to the so-called superpotential. Let us propose in the q-deformed context analogous supplementary terms but by taking into account the different dimensions of Grassmann quantities and the associated covariant derivatives, when $q = 1$ or $q \neq 1$. We thus define the new Lagrangian through

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{D}_1(q)\theta_1 W_1(\Phi) + \mathscr{D}_2(q)\theta_2 W_2(\Phi) \tag{4.1}$$

where we have introduced *two* superpotentials since there is no (*a priori*) reaso privilege θ_1 instead of θ_2 or vice versa. In parallelism with the free case, the funct $\mathscr{D}_1(q)$ and $\mathscr{D}_2(q)$ ensure the self-adjointness of $\mathscr{L}$. These two interacting terms developed following Taylor's formula to obtain

$$W_j(\Phi) = W_j(x) + hW_j'(x) + \tfrac{1}{2}h^2 W_j''(x) + \ldots \qquad j = 1, 2 \qquad ($$

where

$$h=q^{-1/2}\theta_1\psi_1(t)+q^{-1/2}\theta_2\psi_2(t)+q^{-1/2}\theta_1^2A_1(t)+q^{-1/2}\theta_2^2A_2(t)$$
$$+\theta_1\theta_2B(t)+q\theta_1^2\theta_2\chi_1(t)+q^2\theta_2^2\theta_1\chi_2(t)+\theta_1^2\theta_2^2F(t)$$

and primes stand for derivatives with respect to x. We then obtain, taking care of (3.6)

$$\begin{aligned}\mathscr{L}=\mathscr{L}_0+\mathscr{D}_1(q)\theta_1^2\theta_2^2[\chi_2W_1'(x)+\tfrac{1}{2}(1-q)(\psi_1A_2-q^{1/2}\psi_2B)W_1''(x)\\+\tfrac{1}{6}(q^{3/2}\psi_1\psi_2^2+q^{-1/2}\psi_2\psi_1\psi_2+q^{1/2}\psi_2^2\psi_1)W_1'''(x)]\\+\mathscr{D}_2(q)\theta_1^2\theta_2^2[q^2\chi_1W_2'(x)+\tfrac{1}{2}(q-1)(q^{1/2}\psi_1B-A_1\psi_2)W_2''(x)\\+\tfrac{1}{6}(q^{-1/2}\psi_1^2\psi_2+q^{1/2}\psi_1\psi_2\psi_1+q^{3/2}\psi_2\psi_1^2)W_2'''(x)].\end{aligned}\tag{4.3}$$

Applying once again the projection technique with

$$d_1(q)=-\tfrac{1}{4}q\mathscr{D}_1(q)\qquad d_2(q)=-\tfrac{1}{4}\mathscr{D}_2(q)\tag{4.4}$$

and

$$q\psi_1\psi_2=\psi_2\psi_1\tag{4.5}$$

we are led to

$$\begin{aligned}L=L(\dot{x},\psi_j,\dot{\psi}_j,A_j,\dot{A}_j,B,\chi_j,F)\\=L_0+d_1(q)[\chi_2W_1'(x)+\tfrac{1}{2}(1-q)(\psi_1A_2-q^{1/2}\psi_2B)W_1''(x)]\\+d_2(q)[q^2\chi_1W_2'(x)+\tfrac{1}{2}(q-1)(q^2A_1\psi_2+q^{-1/2}\psi_1B)W_2''(x)].\end{aligned}\tag{4.6}$$

The result (3.8) on the auxiliary fields B and F is still true and the relations (3.9) are now written

$$\chi_1=-q^{3/2}\dot{x}+\frac{d_2(q)}{C(q)}qW_2'(x)\tag{4.7a}$$

$$\chi_2=-q^{3/2}\dot{x}+\frac{d_1(q)}{C(q)}q^2W_1'(x).\tag{4.7b}$$

Together with (3.11), the corresponding Hamiltonian takes the form

$$\begin{aligned}H=\tfrac{1}{2}\dot{x}^2-\tfrac{1}{2}q^2\frac{d_1^2(q)}{C(q)}W_1'^2(x)-\frac{1}{2}\frac{d_2^2(q)}{C(q)}W_2'^2(x)\\+\tfrac{1}{2}d_1(q)(q-1)\psi_1A_2W_1''(x)+\tfrac{1}{2}d_2(q)(q^2-1)A_1\psi_2W_2''(x).\end{aligned}\tag{4.8}$$

The self-adjointness condition on H holds if

$$d_1(q)=\alpha_1+\mathrm{i}\beta_1(q-q^2)\qquad d_2(q)=\alpha_2q+\mathrm{i}\beta_2(1-q^2)\tag{4.9}$$

α_j, β_j ($j=1,2$) being real numbers. The information (4.8) is also completed by

$$C^{-1}(q)=2q.\tag{4.10}$$

Let us now go to quantization. Besides the result (3.18), we notice that the relations (3.6) are compatible with

$$\psi_1^6=\psi_2^6=A_1^6=A_2^6=I.\tag{4.11}$$

However, if we choose $d_1(q)=0$, we can identify A_1 and ψ_2 with the Pauli matrices σ_1 and σ_2, respectively (see (3.6*a*), and (4.11) becoming simply $A_1^2=\psi_2^2=I$). In this case, the quantized version of the operator (4.8) reduces to

$$H=\frac{2q}{(1+q)^2}p^2-\frac{1}{2}\,\frac{d_2^2(q)}{C(q)}\,W_2'^2(x)+\frac{i}{2}\,d_2(q)(q^2-1)\sigma_3 W_2''(x) \tag{4.12}$$

or, with (4.9) and (4.10), to

$$H=\frac{2q}{(1+q)^2}p^2+[(q+q^2)(\alpha_2^2+3\beta_2^2)-(2i(q^2-q))\alpha_2\beta_2]W_2'^2(x)$$
$$+\frac{i}{2}(\alpha_2(1-q)-3i\beta_2(1+q))\sigma_3 W_2''(x). \tag{4.13}$$

Let us fix α_2 and β_2 such that H goes to the Witten Hamiltonian [5] when $q\to 1$. This leads to

$$\alpha_2=\sqrt{\tfrac{1}{6}} \qquad \beta_2=\tfrac{1}{6}. \tag{4.14}$$

The q-supersymmetric Witten Hamiltonian is finally given in the form

$$H=\frac{2q}{(1+q)^2}p^2+\frac{1}{2}\left(\frac{1}{2}(q+q^2)-\frac{2i}{3\sqrt{6}}(q^2-q)\right)W_2'^2(x)$$
$$+\frac{1}{2}\left(\frac{1}{2}(1+q)+\frac{i}{\sqrt{6}}(1-q)\right)\sigma_3 W_2''(x). \tag{4.15}$$

5. Comments

We have thus constructed a q-deformed version of the supersymmetric Witten Hamiltonian, by making use of a q-superspace formulation when $q^3=1$. In this way, our approach essentially differs from a previous analysis [14] (purely quantum and based on real q) of the problem and takes into account specific features of self-adjointness of both operators x and p, in contrast to the former one. However, we have to mention that our point of view is not unique due to different possible choices concerning the relations (2.9), for example. Let us also add, in the same spirit, that one could generalize the quantization of (3.12) by asking for q in (3.15) not to be necessarily the same one as in (2.1).

Let us finally notice that our developments are relevant to a fractional ($M=3$, $F=3$) superspace [15] as is clear from (2.1) and the dimension of the deformed Grassmann quantities. One can also be convinced of such a fact by noticing that the Lagrangians (3.1) and (4.1) are invariant under the q-supersymmetric transformations

$$\theta_1'=\theta_1-q^{3/2}\varepsilon_1 \tag{5.1a}$$

$$\theta_2'=\theta_2-q^{3/2}\varepsilon_2 \tag{5.1b}$$

$$t'=t+q\varepsilon_1\theta_1^2+q\varepsilon_2\theta_2^2-q^{-1/2}\varepsilon_1^2\theta_1-q^{-1/2}\varepsilon_2^2\theta_2 \tag{5.1c}$$

compatible with the reality requirements if

$$\theta_1\varepsilon_1=q\varepsilon_1\theta_1 \qquad \theta_2\varepsilon_2=q\varepsilon_2\theta_2 \qquad \theta_1\varepsilon_2=q\varepsilon_2\theta_1 \qquad \theta_2\varepsilon_1=q^2\varepsilon_1\theta_2 \tag{5.2}$$

and

$$\varepsilon_1\varepsilon_2=q\varepsilon_2\varepsilon_1. \tag{5.3}$$

The transformations lead to

$$\Phi'=\Phi+\mathrm{i}q\varepsilon_1 Q_1\Phi+\mathrm{i}q\varepsilon_2 Q_2\Phi+\varepsilon_1^2Q_1^2\Phi+\varepsilon_2^2Q_2^2\Phi-q\varepsilon_1\varepsilon_2Q_1Q_2\Phi$$
$$+\mathrm{i}q^2\varepsilon_1^2\varepsilon_2Q_1^2Q_2\Phi+\mathrm{i}\varepsilon_2^2\varepsilon_1Q_2^2Q_1\Phi+q^2\varepsilon_1^2\varepsilon_2^2Q_1^2Q_2^2\Phi \tag{5.4}$$

where

$$Q_j=\partial_{\theta_j}-\mathrm{i}\theta_j^2\partial_t \qquad j=1,2. \tag{5.5}$$

One should not be surprised about the coefficients in (5.4), coming from a deformation [16] of the Baker–Campbell–Hausdorff formula, in order to keep self-adjointness. We then find the relations

$$Q_1^3=Q_2^3=-H \qquad [H,Q_1]=[H,Q_2]=0 \qquad Q_1Q_2=qQ_2Q_1$$

typical of a fractional superspace.

Let us conclude with possible extensions of the present work. It is indeed natural to wonder what the possible spectra subtended by (4.15) are, if there is a double degeneracy as in the usual supersymmetric quantum mechanics, what the eigenfunctions of (4.15) are (q-Hermite polynomials), etc. We plan to come back to these questions.

Acknowledgments

The author expresses sincere gratitude to Professors J Beckers and J de Azcarraga for interesting discussions. Thanks are also due to Professor J Beckers for a careful reading of the manuscript and to the referee for constructive comments.

References

[1] Lahiri A, Roy P K and Bagchi B 1990 *Int. J. Mod. Phys.* A **5** 1383
[2] Fayet P and Ferrara S 1977 *Phys. Rep.* C **32** 250
[3] Lancaster D 1984 *Nuovo Cimento* **79A** 28
[4] Berezin F A 1966 *The Method of Second Quantization* (New York: Academic)
[5] Witten E 1981 *Nucl. Phys.* B **188** 513
[6] Drinfeld V G 1986 *Quantum groups*, in *Proceedings of the International Congress of Mathematicians*, Berkeley
[7] Majid S 1990 *Int. J. Mod. Phys.* A **5** 1
[8] Filippov A T, Isaev A P and Kurdikov A B 1992 *Mod. Phys. Lett.* A **7** 2129
[9] Kerner R 1992 *J. Math. Phys.* **33** 403
[10] Matheus-Valle J L and Monteiro M A R 1992 *Mod. Phys. Lett.* A **7** 3023
[11] Müller-Kirsten H J W and Wiedemann A 1987 *Supersymmetry. An Introduction with Conceptual and Calculational Details* (Singapore: World Scientific)
[12] Macfarlane A J 1989 *J. Phys. A: Math. Gen.* **22** 4581
[13] Aref'eva I Ya and Volovich I V 1991 *Phys. Lett.* **268B** 179
Li Y Q and Sheng Z M 1992 *J. Phys. A: Math. Gen.* **25** 6779
Ubriaco M R 1993 *Mod. Phys. Lett.* A **8** 89
[14] Spiridonov V P 1992 *Mod. Phys. Lett.* A **7** 1241
[15] Durand S 1992 *Mod. Phys. Lett.* A **7** 2905
[16] Katriel J and Solomon A I 1991 *J. Phys. A: Math. Gen.* **24** L1139